OVERALL VOLTAGE GAIN OF LOW FREQUENCY AMPLIFIERS WITH NEGATIVE RESISTANCE*

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ABSTRACT. A detailed study of the low-frequency amplifiers has been made to investigate the loss of amplified voltage while it is transferred from one stage to the other. An equivalent circuit of a resistance-capacity-coupled amplifier has been analysed and it is found that the loss of voltage will be minimum with a negative grid resistance the value of which depends on the circuit elements. It is also pointed out that the maximum value of the gain increases for high values of the coupling condenser. The above conclusions are verified by using the negative resistance of a dynatron valve in the grid circuit of the valve in the second stage of the amplifier.

INTRODUCTION

The overall voltage gain of a low-frequency valve amplifier of more than one stage is generally found to be sensibly less than that calculated theoretically from the constants of the valves used and the circuit components connected with them. This disagreement has often been attributed partly to the stray and fixed capacities and inductances involved in the circuit, and partly to the behaviour of the valve under the circumstances.[1,2,3]

In the present communication a detailed study has been made of low-frequency amplifiers investigating the cause of the loss of amplified voltage when the energy is transferred from one valve to the next following it. A preliminary note on this has already been published.[4] A two-stage low-frequency resistance-capacity-coupled amplifier was employed for the present investigation and it has been found after analysis of such circuit that the primary seat of attenuation of voltage transferred from one stage to the other lies in the intervalve coupling components. Out of all the components the grid resistance of the second valve only can be varied at will for compensating the loss of voltage incurred without causing any serious alteration in the circuit or valve parameters. The minimum loss of voltage is, however, obtained only by a negative resistance in the grid circuit of the second valve, the exact value of which has been calculated mathematically as shown subsequently. The condition of minimum loss has been verified experimentally by applying a negative resistance with a dynatron valve along

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with a fixed positive resistance in parallel with it in the grid circuit of the second valve. The resistance of the dynatron was varied from positive to negative values by changing the grid and plate potentials applied, and the gradual change of the overall amplification passing through the maximum value has been recorded.

THEORY

The equivalent circuit diagram of a resistance-capacity-coupled low-frequency amplifier is shown in fig. 1. R_1 represents the plate resistance of the first valve

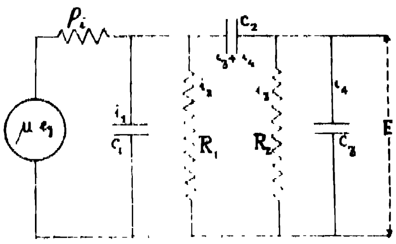

FIG. 1

and R_2 represents the grid resistance of the second valve. C_1 and C_3 are the inter-electrode capacities of the valves. C_2 is the coupling condenser. ρ represents the internal resistance of the first value V_1. Let i_1, i_2, i_3 and i_4 be the currents flowing through different branches of the circuit and i the total current as shown in fig. 1.

If w be the angular frequency of the input e.m.f., the following circuit equations can be obtained from fig. 1.

$$i = i_1 + i_2 + i_3 + i_4 \quad \dots \quad (1)$$

$$\mu e_g = \rho i + \frac{i_1}{jwC_1} \quad \dots \quad (2)$$

$$R_1 i_2 = \frac{i_1}{jwC_1} \quad \dots \quad (3)$$

$$R_2 i_3 = \quad \dots \quad (4)$$

$$R_1 i_2 = \frac{i_3 + i_4}{jwC_2} + R_2 i_3. \quad \dots \quad (5)$$

If the value of i_2 obtained from equation (5) be substituted in equation (3) and i_4 be expressed in terms of i_3, we will get

$$i_1 = \frac{C_1}{C_2}(i_3 + jwC_3R_2i_3) + jwC_1R_2i_3. \quad \dots \quad (6)$$

From equations (1) and (2) we get after some simplifications

$$\mu e_g = \rho\left\{ \frac{C_1}{C_2}\,(i_3 + jwC_3R_2i_2) + jwC_1R_2i_3 \right\}$$

$$+ \frac{\rho}{R_1}\left\{ \frac{1}{jwC_2}\,(i_3 + jwC_3R_2i_3) + R_2i_3 \right\}$$

$$+ \rho i_3 + jwC_3\rho R_2 i_3 + \frac{1}{jwC_2}\,(i_3 + jwC_3R_2i_3) + R_2i_3. \quad \ldots \quad (7)$$

Taking the output voltage E to be equal to R_2i_3 and eliminating the currents we can rewrite the above equation (7) as

$$\mu e_g = E\left[\left\{ 1 + \frac{\rho C_1}{R_2C_2} + \frac{\rho C_3}{R_1C_2} + \frac{\rho}{R_1} + \frac{\rho}{R_2} + \frac{C_3}{C_2} \right\} + j\left\{ \frac{w\rho C_1C_3}{C_2} + w\rho C_1 - \frac{\rho}{wC_2R_1R_2} + \rho wC_3 - \frac{1}{wC_2R_2} \right\}\right]. \quad \ldots \quad (8)$$

From equation (8) the absolute value of E will be given by

$$|E| = \frac{\mu e_g}{\sqrt{\left(A + \frac{B}{R_2}\right)^2 + \left(C - \frac{D}{R_2}\right)^2}} \quad \ldots \quad (9)$$

where $$A = 1 + \frac{C_3}{C_2} + \frac{\rho}{R_1} + \frac{\rho C_3}{R_1C_2}$$

$$B = \frac{\rho C_1}{C_2} + \rho$$

$$C = \frac{w\rho C_1C_3}{C_2} + wC_1\rho + wC_3\rho$$

$$D = \frac{\rho}{wR_1C_2} + \frac{1}{wC_2}.$$

It may be pointed out that in order to obtain the maximum value of E, the voltage developed across the grid and the filament of the second valve, the plate resistance R_1 cannot be changed to a large extent as it will alter the working conditions of the valve. The loss of transferred voltage decreases with the increase of the capacity of the coupling condenser C_2 and it will be minimum when the capacity reaches infinity. But this is not practicable as it will make the grid of the second valve highly positive and the valve will cease to function.

Therefore in order to get the maximum value of $|E|$ we differentiate the equation (9) with respect to grid resistance R_2 and get

$$\frac{d|E|}{dR_2} = -\mu e_g \left\{ \left(A + B/R_2\right)^2 + \left(C - D/R_2\right)^2 \right\}^{-3/2}$$

$$\left\{ \frac{D}{R_2^2}\left(C - D/R_2\right) - \frac{B}{R_2^2}\left(A + B/R_2\right) \right\}. \quad \ldots \quad (10)$$

Equating the expression for $\frac{d|E|}{dR_2}$ to zero, we get the condition for maximum or minimum value of transferred voltage given by

$$R_2 = \frac{B^2 + D^2}{DC - AB}$$

$$= \frac{\rho_2 + \frac{\rho^2 C_1^2}{C_2^2} + \frac{2\rho^2 C_1}{C_2} + \frac{\rho^2}{w^2 R_1^2 C_2^2} + \frac{1}{w^2 C_2^2} + \frac{2\rho}{w^2 R_1 C_2^2}}{-\left(\rho + \frac{\rho^2}{R_1}\right)}. \quad \ldots \quad (11)$$

Differentiating $|E|$ once more with respect to R_2 we have

$$\frac{d^2|E|}{dR_2^2} = \mu e_g \left\{ \left(A + B/R_2\right)^2 + \left(C - D/R_2\right)^2 \right\}^{-5/2}$$

$$\left[3\left\{ \frac{D}{R_2^2}\left(C - D/R_2\right) - \frac{B}{R_2^2}\left(A + B/R_2\right) \right\}^2 \right.$$

$$\left. -\left(\frac{3B^2}{R_2^4} - \frac{2DC}{R_2^3} + \frac{2AB}{R_2^3} + \frac{3D^2}{R_2^4} \right) \left\{ (C - D/R_2)^2 + (A + B/R_2)^2 \right\} \right]. \quad (12)$$

Substituting, the value of R_2 from equation (11) in the above equation and simplifying, we get

$$\frac{d^2|E|}{dR_2^2} = -\mu e_g \left\{ \left(A + B/R_2\right)^2 + \left(C - D/R_2\right)^2 \right\}^{-3/2}$$

$$\frac{1}{R_2^4 (B^2 + D^2)} \left(2AB^3CD + A^2B^2D^2 + 2ABCD^3\right.$$

$$\left. + A^2D^4 + B^4C^2 + B^2C^2D^2\right). \quad \ldots \quad (13)$$

From the above equation it will be observed that the value of $\frac{d^2|E|}{dR_2^2}$ is negative. Thus the expression for R_2 given in equation (11) corresponds to the maximum value of the transferred voltage.

From equation (11) it will be observed that the resistance required for minimum loss of voltage will be always negative and the magnitude of this will depend on the constants of the valve used and the values of other associated components. This condition for maximum transfer of voltage has been verified by applying the negative resistance of a dynatron valve between the grid and the filament circuit of the second valve of the amplifier as shown in the next section.

Two typical curves have been drawn in fig. 2, to indicate the variation of the voltage gain in decibels when the grid resistance is altered from positive to negative values. The voltage gain is calculated from equation (9). Curve A in fig. 2 has been drawn for high value of coupling condensers and B for low value of the same. From these curves it will be observed that the magnitude of the negative resistance for maximum transfer of voltage decreases as the capacity of the coupling condenser is increased. It will finally reach a limit of about 10,000 ohms with very large value of coupling condensers. It will be further observed that the maximum value of the voltage gain increases very rapidly as the capacity of the coupling condenser is increased. This is shown by the peak of the curve A at P. It may be noted that under suitable circumstances voltage gain with negative resistance may be substantially higher than that with positive resistance. The portions of the curves for negative voltage gain have not been shown in fig. 2, but they can be easily computed from equation (11).

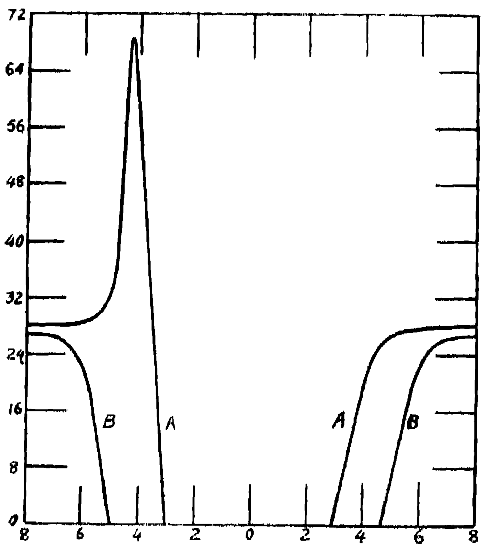

FIG. 2

Experimental Arrangement and Observations

The connections were made as shown in fig. 3 above. V_1 and V_2 represent the two valves of different stages of the amplifier. Other notations for different components are retained the same as in fig. 1. The overall amplification is obtained by measuring the amplified voltage E across the grid of the second valve by a thermionic voltmeter. A small input voltage was applied between the grid and the filament of the first valve from an audio-frequency valve oscillator

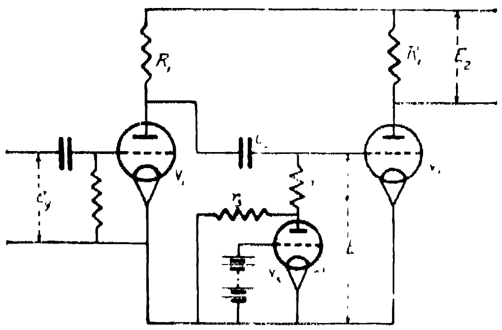

FIG. 3

through an attenuator. Different values of positive and negative resistances were applied in the grid circuit by a dynatron valve V_3 of internal resistance r_1 along with two other resistances r_2 and r_3. Thus the combination of r_1, r_2 and r_3 is equivalent to R_2 in fig. 1. The values of the positive and negative resistances of the valve were obtained from its characteristics drawn under similar conditions.

Table I below gives the observed and calculated values of $|E|/e_g$ for different negative grid leak resistances R_2 of the second valve, with the following circuit constants :

$\rho = 3{,}25{,}000$ ohms,

$\mu = 25$,

$C_2 = .002$ microfarad,

$C_1 = C_3 = 10$ micro-microfarads,

$R_1 = 2{,}50{,}000$ ohms,

R_2 for minimum loss of voltage $= -1.87 \times 10^5$ ohms.

TABLE 1

Negative grid leak resistance (R_2) in ohms	Voltage gain (calculated) in decibels	Voltage gain (observed) in decibels
7.1	22.5	21.7
6.0	22.8	22.1
5.0	23.3	22.7
4.1	23.9	23.5
3.5	24.5	24.7
3.0	25.1	25.5
2.8	25.4	25.6
2.1	26.5	26.1
1.9	26.6	26.4
1.5	24.9	26.0
0.5	8.9	11.1

Fig. 4 is drawn to show the variation of voltage gain with negative values

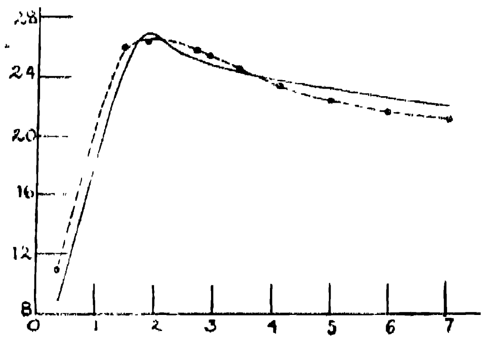

FIG. 4

of R_2. The continuous curve is drawn from the theoretically calculated values and dotted one from experimentally observed values.

SUMMARY

Analysis of the circuit of a low-frequency resistance-capacity-coupled amplifier has shown that the loss of voltage incurred in transferring the energy from one stage to the other is minimum with negative resistance in the grid circuit of

the second stage of the amplifier. The value of the negative resistance required for minimum loss has been calculated theoretically from the equations derived for the purpose and it has been experimentally verified by inserting a dynatron valve in the grid circuit. It has been shown that the maximum voltage gain with negative resistance is always higher than the gain with positive resistance.

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